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Heat transfer in full-scale coolers of rubber

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Abstract—Intensity of cooling was measured on three commercial coolers of rubber. The rate of cooling was interpreted in terms of the overall heat transfer coefficient which was determined with the aid of a simplified, one-dimensional model of unsteady-state heat transfer.

INTRODUCTION

THE TEMPERATURE of a rubber band increases considerably in the course of its mechanical treatment. In order to avoid the spontaneous vulcanization, the efficient and economical cooling of a rubber material is necessary.

The rate of heat transfer from the treated band of rubber depends generally on its physical properties such as the thermal conductivity, heat capacity, density and its thickness, and on the intensity of cooling. The intensity of cooling is given by the temperature difference between the interface and the bulk of cooling fluid, and by the heat transfer coefficient. The magnitude of the heat transfer coefficient depends predominantly on complex phenomena occurring at the interface. By its nature the heat transfer coefficient is an empirical quantity. Only in well-defined situations can its magnitude be predicted from correlations available in the literature. In specific cases it has to be determined by experiment. The experience shows that the heat transfer coefficient is usually effected by the manner in which its determination is performed.

The aim of this work is to develop a procedure that makes it possible to evaluate the intensity of rubber cooling in commercial apparatus.

THEORY

The cooling of a wide rubber band can be treated as a one-dimensional problem. Unsteady heat conduction in a semi-finite slab is considered here for a homogeneous isotropic whose physico-chemical properties are independent of temperature. The asymmetrical cooling of a wide band is described by equation (1)

$$\alpha \frac{\partial^2 v}{\partial x^2} = \frac{\partial v}{\partial t} \tag{1a}$$

for $x \in \langle 0, x_L \rangle$; $t \in \langle 0, \infty)$ and $v = v(x, t)$. The initial and boundary conditions are

$$v(x, 0) = T_0(x) \quad \text{for} \quad x \in \langle 0, x_L \rangle$$

$$k \frac{\partial v}{\partial x} - h_1(t)[v - v_1(t)] = 0 \quad \text{at} \quad x = 0, \quad t \in \langle 0, \infty) \tag{1b}$$

$$k \frac{\partial v}{\partial x} + h_2(t)[v - v_2(t)] = 0 \quad \text{at} \quad x = x_L;\ t \in \langle 0, \infty). \tag{1c}$$

In general, quantities h_1, h_2, v_1 and v_2 can vary with time and are considered in the general formulation as functions of t.

In the particular case, when $T_0(x) = \text{const.}$ for $x \in \langle 0, x_L \rangle$ and the parameters h_1, h_2, v_1 and v_2 are constant in time, the analytical solution of (1) can be obtained with the aid of a series. This solution is presented in a previous work of the authors [1].

A commercial cooling apparatus usually consists of several sections, each with different operating conditions as shown in Fig. 1. The analytical solution cannot, therefore, be employed for describing the entire cooling process. The parameters h_1, h_2, v_1 and v_2 vary in the course of the cooling. The temperature profile at the exit of section A is the initial profile for the next section B. As can be seen the assumption $T_0(x) = \text{const.}$, $x \in \langle 0, x_L \rangle$ which is necessary for the analytical solution, cannot be made.

In order to describe the cooling of a band under such practical conditions, equation (1) was solved by a finite-difference technique. The following approximations were employed for differentiation of the set (1):

$$v(x_i, t_j) = v_{i,j}$$

for

$$x_i = (i-1)\Delta x; \quad i = 1, \ldots, N; \ \Delta x = x_L/(N-1)$$

$$t_j = (j-1)\Delta t; \quad j = 1, \ldots, M; \ \Delta t = t/(M-1),$$

where t is the time at which the value $v(x, t)$ is sought.

$$\frac{\partial v}{\partial t} \doteq \frac{v_{i,j} - v_{i,j-1}}{\Delta t}$$

$$\frac{\partial^2 v}{\partial x^2} \doteq \frac{v_{i+1,j} - 2v_{i,j} + v_{i-1,j}}{(\Delta x)^2}$$

$$\frac{\partial v}{\partial x} \doteq \frac{v_{2,j} - v_{1,j}}{\Delta x} \quad \text{at} \quad x = x_1;\ t = t_j$$

$$\frac{\partial v}{\partial x} \doteq \frac{v_{N,j} - v_{N-1,j}}{\Delta x} \quad \text{at} \quad x = x_N = x_L;\ t = t_j. \tag{2}$$

On substituting the above approximations in

NOMENCLATURE

- C_P heat capacity at constant pressure [J kg^{-1} K^{-1}]
- h heat transfer coefficient [W m^{-2} K^{-1}]
- h_1 heat transfer coefficient at $x \to -0$ [W m^{-2} K^{-1}]
- h_2 heat transfer coefficient at $x \to x_L+$ [W m^{-2} K^{-1}]
- h_a heat transfer coefficient in the drying section [W m^{-2} K^{-1}]
- h_w heat transfer coefficient in the spray section [W m^{-2} K^{-1}]
- k thermal conductivity of rubber material [W m^{-1} K^{-1}]
- K number of temperature measurements
- M number of time steps
- M_k temperature measured at the center plane of slab in time t_k [K]
- N number of spatial steps
- n number of interval halving
- $S(h)$ function defined by equation (7)
- t time [s]
- t_w exposure time in the spray section [s]
- T_0 initial temperature of slab [K]
- v temperature of slab [K]
- v_1 temperature of cooling medium at $x \to -0$ [K]
- v_2 temperature of cooling medium at $x \to x_L+$ [K]
- $\bar{v}_a$ mean temperature of cooling air in the drying section [K]
- $\bar{v}_w$ mean temperature of cooling water in the spray section [K]
- v_{a1} temperature of entering air [K]
- v_{a2} temperature of outgoing air [K]
- v_{w1} temperature of entering water
- v_{w2} temperature of outgoing water
- x coordinate [m]
- x_L thickness of slab [m].

Greek symbols

- $\alpha \equiv k/(\rho C_P)$ thermal diffusivity of slab [m^2 s^{-1}]
- ρ density [kg m^{-3}].

Dimensionless groups

- Bi Biot number, $hx_L/(2k)$.

equation (1) we can write the following difference equations

$$\frac{v_{i,j}-v_{i,j-1}}{\Delta t} = \alpha\frac{v_{i+1,j}-2v_{i,j}+v_{i-1,j}}{(\Delta x)^2}$$

where $i = 2,\dots,N-1$

$$k\frac{v_{2,j}-v_{1,j}}{\Delta x} - h_1(v_{1,j}-v1_j) = 0; \quad \text{at} \quad i = 1$$

$$k\frac{v_{N,j}-v_{N-1,j}}{\Delta x} + h_2(v_{N,j}-v2_j) = 0; \quad \text{at} \quad i = N \quad (3)$$

where $v1_j$ and $v2_j$ are the external temperatures $v_1(t)$ and $v_2(t)$ at times t_j.

On rearranging equations (3) we get, for the desired values $v_{i,j}$, the following set of linear algebraic equations:

$$v_{1,j}\cdot\left(-\frac{k}{\Delta x}-h_1\right)+v_{2,j}\cdot\frac{k}{\Delta x} = -h_1\cdot v1_j \quad \text{for} \quad i = 1$$

$$v_{i-1,j}\cdot\frac{\alpha}{(\Delta x)^2}+v_{i,j}\cdot\left(-\frac{2\alpha}{(\Delta x)^2}-\frac{1}{\Delta t}\right)+v_{i+1,j}\cdot\frac{\alpha}{(\Delta x)^2} = -\frac{v_{i,j-1}}{\Delta t} \quad \text{for} \quad i = 2,\dots,N-1 \quad (4)$$

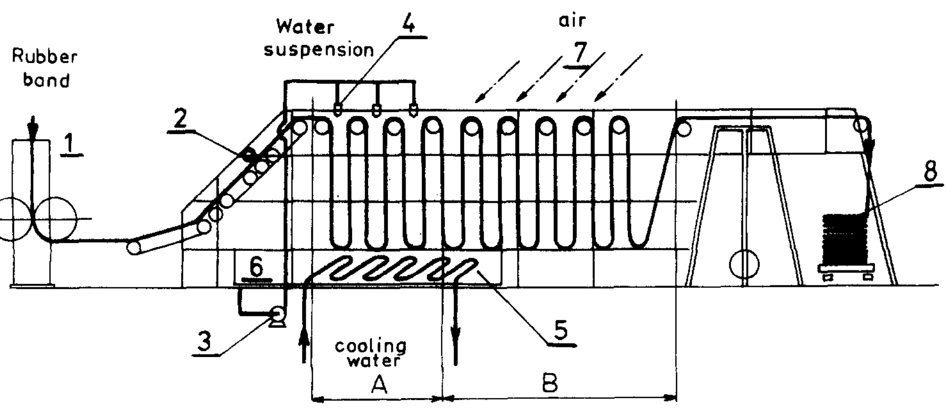

FIG. 1. Schematic diagram of the cooling apparatus: 1, calender; 2, cutter; 3, circulation pump; 4, spray nozzle; 5, suspension cooler; 6, suspension tank; 7, drying air; 8, folding apparatus; A, spray section; B, drying section.

$$v_{N-1,j}\cdot\left(-\frac{k}{\Delta x}\right)+v_{N,j}\cdot\left(\frac{k}{\Delta x}+h_2\right)=h_2\cdot v2_j \quad \text{for} \quad i=N.$$

The set (4) is solved with respect to the variable $v_{i,j}$ on substituting the known values $v_{i,j-1}$. The computation proceeds by lines successively for $j = 2,\ldots,M$. The values $v_{i,1}$ are given by the initial condition, i.e. $v_{i,1} = T_0(x_i)$.

A subprogram was written that enables one to find the solution for an arbitrary initial profile of temperature $T_0(x)$ and for the parameters h_1, h_2, v_1 and v_2 that vary with the time t. In an actual computation, the numbers of grid points were chosen as $N = 20$ and $M = 50$. The comparison of the numerical and analytical solutions for the case when $T_0 = \text{const.}$ showed that the solutions differ in the fourth digit. At temperatures of ~ 300 K, the difference between both solutions was less than 0.6 K. The amount of computer time is about the same for both procedures.

As can be seen the above model and its solution can easily be combined with an optimization procedure. Then the heat transfer coefficients can be determined on the basis of measured temperatures within the band at different elapsed times of cooling.

APPLICATION OF THE MODEL TO COMMERCIAL COOLERS OF RUBBER

The function of a commercial cooler was approximated with the aid of model equations (1a)–(1c) using the following assumptions:

1. The band goes through the spray section at first, then passes through the drying section.
2. Intensity of cooling is the same on both sides of the band, i.e. $v_1 = v_2$ and $h_1 = h_2$.
3. Heat transfer coefficients and temperatures of cooling media in the sections are represented by the mean values of these quantities, i.e. also with respect to the symmetry,

$$h_1(t) = h_2(t) \doteq h_w \quad \text{at} \quad t \leqslant t_w$$
$$h_1(t) = h_2(t) \doteq h_a \quad \text{at} \quad t > t_w \tag{5}$$
$$v_1(t) = v_2(t) \doteq \bar{v}_w = \tfrac{1}{2}(v_{w1}+v_{w2}) \quad \text{at} \quad t \leqslant t_w$$
$$v_1(t) = v_2(t) \doteq \bar{v}_a = \tfrac{1}{2}(v_{a1}+v_{a2}) \quad \text{at} \quad t > t_w \tag{6}$$

4. The increase in temperature of the cooling media brought about by passing through the cooler is not large and the mean temperature of a medium is lower than that of the band.

To determine the course of cooling, the center plane temperatures were measured as those at the depth equal to the half thickness of the band at different times of cooling. The heat transfer coefficient was estimated on the basis of these experimental results using the following simple optimization procedure.

At the start, an interval is set in which the sought value of h_w (h_a) is likely to occur. In this interval, the minimum is searched for the function

$$S(h) = \sum_{k=1}^{K} [v(x_L/2, t_K) - M_k]^2 \tag{7}$$

where M_k are the values of temperature measured in the middle of the band at the elapsed times of cooling t_k, $k = 1,\ldots,K$. The function $v(x, t)$, which is the solution of equations (1a)–(1c), depends generally on the values of the parameters h, v_1 and v_2. When the values v_1 and v_2 are known, we can estimate the remaining parameter h from the condition for an extreme of the function

$$\frac{dS(h)}{dh} = 0. \tag{8}$$

If the function S has a single extreme in the chosen interval and this extreme is a minimum, then the problem is reduced to finding the single root of equation (8).

The derivative in equation (8) is evaluated numerically and the equation can easily be solved by interval halving. When the basic interval is chosen to be as large as $\langle 0, 1000\rangle$ or $\langle 0, 100\rangle$, the quantity h is found after tenfold halving with accuracy better than 1.0 or 0.1 W m^{-2} K^{-1}, respectively.

EXPERIMENTAL SECTION

The temperatures at the center plane of a band were measured at five positions in the cooler by rapid thermocouples with accuracy ± 0.5°C. All measurements were repeated five times and the averaged values of temperature were used in the subsequent treatment of data. The temperatures of cooling media were measured by thermometers with accuracy ± 0.2°C. The rate of band movement was determined by means of a stop-watch. The physical parameters of rubber materials were provided by the manufacturers and are summarized in Table 1.

The measurements were conducted on three commercial, full-scale coolers under normal operating conditions—no planned encroaches on the running conditions could be made. The production capacity of the coolers ranged from 5000 to 7000 kg h^{-1}. Every effort was taken to avoid possible errors brought about by gross external disturbances.

RESULTS

Temperatures of the cooling media entering and leaving the coolers are summarized in Table 2. The temperature increases of the coolants vary from 2.2 to 4.4 K. From the measured cooling curves of rubber bands, the heat transfer coefficients in equation 1(c) were determined with the aid of the above optimizing technique and the assumptions (5) and (6). The computed heat transfer coefficients are presented in Table 3.

Agreement between the measured cooling curves and

Table 1. Physical properties of rubber materials in the coolers

Quantity	Symbol	Dimension	Value		
			Cooler I	Cooler II	Cooler III
Density	ρ	$kg\,m^{-3}$	1110.0	1170.0	1150.0
Thermal conductivity	k	$W\,m^{-1}\,K^{-1}$	0.2791	0.2780	0.2733
Heat capacity	C_P	$J\,kg^{-1}\,K^{-1}$	1499.0	1528.0	1516.0
Thermal diffusivity	α	$m^2\,s^{-1}$	1.677×10^{-7}	1.555×10^{-7}	1.568×10^{-7}
Thickness of band	x_L	m	0.010	0.012	0.012
Width of band		m	0.60	0.60	0.65

Table 2. Average temperatures of the cooling media

Cooler	Emulsion (K) Inlet	Emulsion (K) Outlet	Air (K) Inlet	Air (K) Outlet
I	300.4	304.2	298.4	302.7
II	303.4	306.1	302.9	305.1
III	306.1	310.5	297.9	302.3

those computed with the use of determined heat transfer coefficients is explored in Figs. 2–4. Each cooler is represented by one run in these figures. While the cooling is very rapid in the first tens of seconds of exposure to the cooling suspension, the cooling curve is quite flat in the drying section. It can be seen that the computed temperatures are in fair agreement with the experimental values.

It should be noted that the computed temperature profile in the band at the exit of the spray section is used as an initial profile for the drying section. It follows then that the quality of the heat transfer coefficient in the drying section h_a is influenced by that of the heat transfer coefficient h_w in the preceding section.

Values of the heat transfer coefficient in the spray section vary from 92.8 to 302.0 $W\,m^{-2}\,K^{-1}$. The heat transfer coefficients in the drying sections range from 4.5 to 33.7 $W\,m^{-2}\,K^{-1}$. Computational tests showed a considerable sensitivity of the heat transfer coefficients not only to the temperatures, but also to other input quantities. This fact apparently affects the span of the respective heat transfer coefficients.

A physical image of the drying section is substantially simpler than the situation in the spray section. In the drying section the band is cooled by air under conditions of the forced convection. Evaporation of the surface moisture occurs simultaneously. Any appreciable evaporation tends to increase the heat transfer coefficient. The computational estimation of the heat transfer coefficient in the transition flow region according to the literature [2, 3] provides (for an assumed linear air velocity of 5 $m\,s^{-1}$) a value as large as 20 $W\,m^{-2}\,K^{-1}$. Except for Cooler II, the experimental heat transfer coefficients h_a appear to be realistic in light of this predicted value.

The operating conditions in the spray section are so complicated that no closer analysis of heat transfer can be made for this section. An important role is played by the fact that the coolant is not distributed uniformly and that the continuous cooling film does not form on the entire surface of the band. In addition to the heat transport to the falling film of coolant, evaporation and heat transfer to the streaming air also, apparently, occur. Values of the determined heat transfer coefficients in the spray section of coolers range from 93 to 302 $W\,m^{-2}\,K^{-1}$. On average, they are about five times larger than the heat transfer coefficients in the drying section. The above values support the idea that the operating conditions in the spray section are similar to those in trickle coolers used in the heavy chemical industries. The trickle coolers usually consist of a bank of pipes over which water trickles downward, partly evaporating as it travels. For such a cooler Kern [4] estimates the heat transfer coefficient as large as 105 $W\,m^{-2}\,K^{-1}$.

Table 3. Effective heat transfer coefficients in the spray and drying sections of the coolers

Cooler	Run	Heat transfer coefficient in spray section, h_w ($W\,m^{-2}\,K^{-1}$)	Mean temperature of medium, $\bar{v}_w$ (K)	Heat transfer coefficient in drying section, h_a ($W\,m^{-2}\,K^{-1}$)	Mean temperature of medium, $\bar{v}_a$ (K)
I	1	92.8	301.8	22.0	300.8
	2	96.7	301.8	13.2	299.8
	3	92.8	301.8	15.1	299.8
II	1	110.0	303.5	4.4	302.5
	2	128.0	306.0	4.4	305.0
III	1	167.0	307.5	29.8	299.7
	2	302.0	307.7	33.7	300.7

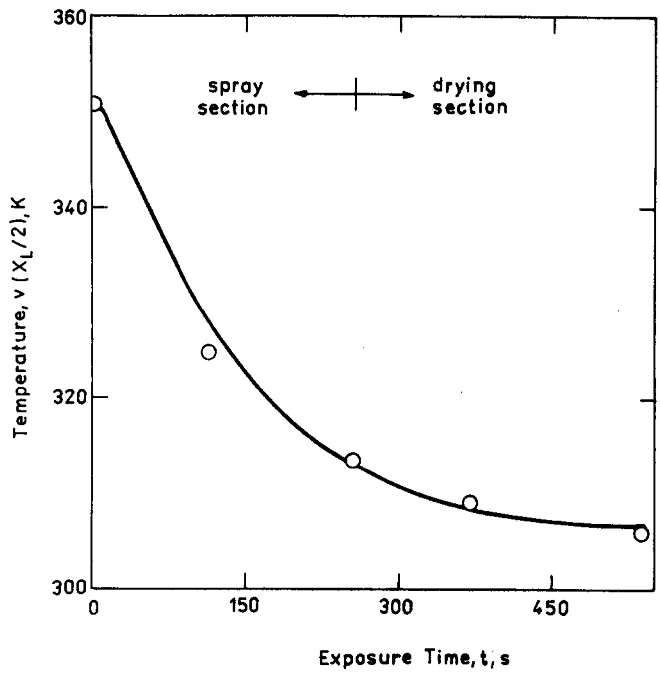

FIG. 2. Dependence of the temperature in the interior of rubber material on the time of cooling: cooler I, run 3; ○ experimental data points. The solid line shows the results computed from equations (1a)–(1c) using the data presented in Tables 1 and 2.

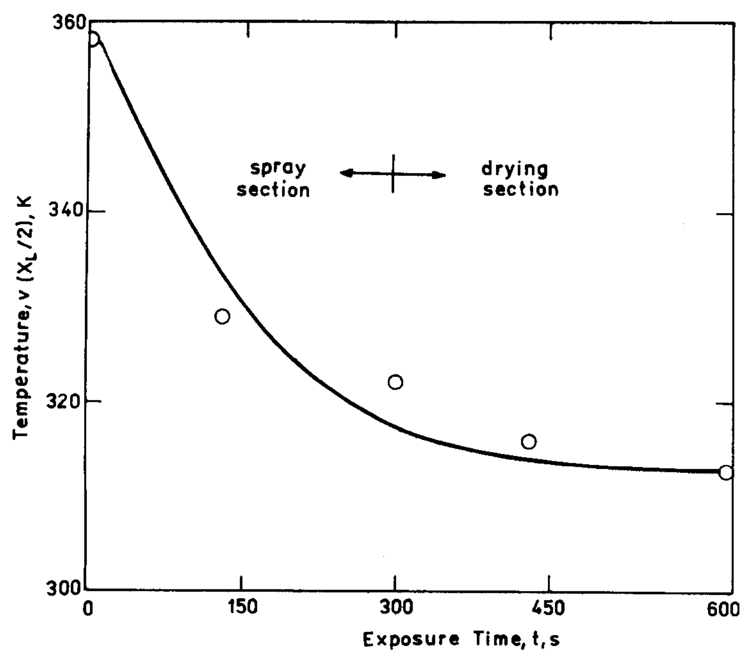

FIG. 3. Dependence of the temperature in the interior of rubber material on the time of cooling: cooler II, run 2; ○ experimental data points. The solid line shows the results computed from equations (1a)–(1c) using the data presented in Tables 1 and 2.

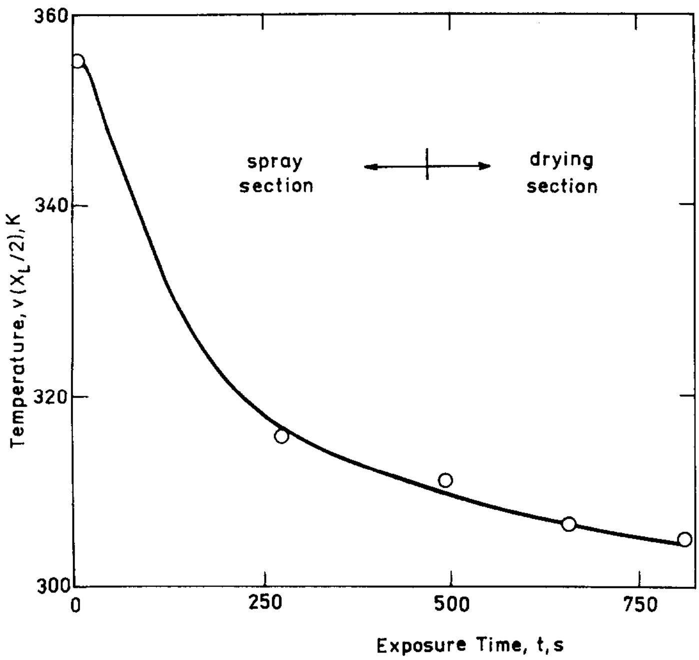

FIG. 4. Dependence of the temperature in the interior of rubber material on the time of cooling: cooler III, run 1; ○ experimental data points. The solid line shows the results computed from equations (1a)–(1c) using the data presented in Tables 1 and 2.

Values of the heat transfer coefficients $h_w = 120$ and $h_a = 20$ W m^{-2} K^{-1} lead to the Biot numbers 3.0 and 0.5, respectively. These values indicate that the dominant resistance is offered in the spray section by heat conduction within the solid and in the drying section by external heat transfer. However, both processes should be considered in analysis of either section. Computations show that the steep temperature profiles develop within the band in the spray section and the flat profiles exist in the drying section as illustrated in Fig. 5.

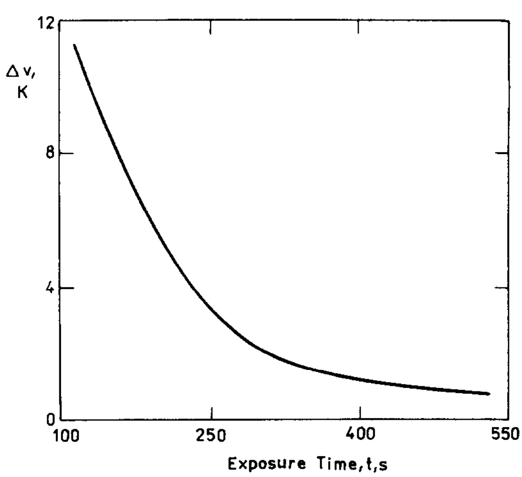

FIG. 5. Dependence of the computed difference between center plane and surface temperature on the time of cooling: cooler I, run 3. The values of model parameters are the same as in Fig. 2.

CONCLUSIONS

The proposed model describes a temperature profile along the length of a commercial cooler with satisfactory accuracy. The heat transfer coefficients evaluated from experimental measurements of temperature are sensitive not only to the changes in temperature profiles, but also to the changes in other model parameters. The determined values of the heat transfer coefficient range from 93 to 302 W m^{-2} K^{-1} in the spray section and vary from 4.5 to 34 W m^{-2} K^{-1} in the drying section.

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TRANSFERT THERMIQUE DANS DES REFROIDISSEURS INDUSTRIELS DE CAOUTCHOUC

Résumé—L'intensité de refroidissement est mesurée dans trois refroidisseurs industriels de caoutchouc. La vitesse de refroidissement est considérée à travers le coefficient global de transfert thermique qui est déterminé à l'aide d'un modèle simple, monodimensionnel de régime thermique permanent.

WÄRMEÜBERTRAGUNG IN KÜHLERN AUS GUMMI

Zusammenfassung—Es wurde die Kühlleistung von handelsüblichen Kühlern aus Gummi gemessen. Die Kühlrate wurde in Form des mittleren Wärmedurchgangskoeffizienten ausgedrückt, welcher mit Hilfe eines vereinfachten eindimensionalen Modells für instationären Wärmeübergang bestimmt wurde.

ТЕПЛООБМЕН В НАТУРНЫХ ОХЛАДИТЕЛЯХ РЕЗИНЫ

Аннотация—Измерялась интенсивность охлаждения на трех промышленных охладителях резины. Темп охлаждения выражался через коэффициент общего теплообмена, который определялся с помощью упрощенной одномерной модели нестационарного теплообмена.